# Summary of Significant Equations for Determining Shadow Length <br> <br> A Reference Guide for Solar Access Evaluations 

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1. Procedure for determining height of shadow up a south wall Formula (see Figure A):
$\mathrm{SH}=\mathrm{H}$ - (Tangent $\theta \times \mathrm{D}$ )
Where
SH= Shadow length of the wall
$\mathrm{H}=$ Height of Object casting shadow
$D=$ Distance from the wall to the obstruction
$\theta=$ Sun Altitude Angle

Example: A 30 -foot tree is located 20 feet due south of the south wall of a passive solar energy system. The land is flat and the site is located at $40^{\circ}$ north latitude. What will be the shadow height on the wall at noon solar time on December $21^{\text {st }}$ ?

The noon altitude angle for December $21^{\text {st }}$ at $40^{\circ}$ north latitude $=26.6^{\circ}$. By substitution into the formula we arrive at the following result:

$$
\begin{aligned}
\text { SH } & =30-\left(\text { TAN } 26.6^{\circ} \times 20\right) \\
& =30-(.501 \times 20) \\
& =30-10.02
\end{aligned}
$$

$S H=19.98$ feet

## 2. Procedure for Determining Height of Shadow up a rooftop

## Formula (See Figure B):

RSH $=(S H-W H) \times$ sine $A_{1}$
sine $A_{2}$

Where:
RSH = Roof shadow height
SH = Shadow height at Wall (determined in equation 1)
WH = Wall Height
$\mathrm{A}_{1}=90^{\circ}$ - Solar Altitude
$\mathrm{A}_{2}=90^{\circ}-\mathrm{A}_{1}+$ roof pitch
Example: A 30-foot tree is located 20 feet due south of the south wall of a dwelling unit with an active solar energy system located on the lowest level of the roof edge. The wall is 8 feet high, the roof has a $20^{\circ}$ degree pitch and the building is located on flat land at $40^{\circ}$ north latitude. What will be the shadow height on the rooftop at noon solar time?

From equation 1 we determined that the shadow height at the wall $=$ 19.98 feet. The noon altitude angle for December $21^{\text {st }}$ at $40^{\circ}$ north latitude $=26.6^{\circ}$.

By substitution into the formula we arrive at the following result:

$$
\begin{aligned}
& \mathrm{A}_{1}=90^{\circ}-26.6^{\circ}=\operatorname{sine} 63.4^{\circ}=.894 \\
& \begin{aligned}
\mathrm{A} 2 & =\left(90^{\circ}-63.4^{\circ}\right)+20^{\circ}=\operatorname{sine} 46.6^{\circ}=.726 \\
\mathrm{RSH} & =(19.98-8) \times(.894 / .726) \\
& =11.98 \times 1.23 \\
& =14.73 \text { feet }
\end{aligned}
\end{aligned}
$$

## 3. Procedure for Determining the Maximum Height Limit for Trees to protect rooftop collectors

Formula (See Figure C):
$H=X+\left(D_{1}+D_{2}\right) \operatorname{Tan} \theta$
Where
H = Height of Object Casting Shadow
$X=$ Height from ground to bottom of collector
$\mathrm{D}_{1}=$ Distance from property line to the collector
$D_{2}=$ Distance from property line to Tree
$\Theta=$ Sun Altitude Angle
Example: A solar collector is located 10 feet above ground level and 20 feet due north of a tree. The land is flat and the site is located at $40^{\circ}$ north latitude. What is the maximum allowable height of a tree at that distance at noon on December $21^{\text {st }}$ ?

The noon altitude angle for December $21^{\text {st }}$ at $40^{\circ}$ north latitude $=26.6^{\circ}$. By substitution into the formula we arrive at the following result:
$H=(10)+(20)$ TAN $26.6^{\circ}$
$H=(10)+(20) \times .501$
$H=30 \times .501=15.03$ feet

Figure A: Minimum Clearance for obstructions for no effect on South Wall Solar Access

SH=H-(Tangent $\theta \times \mathrm{D}$ )
$\mathrm{SH}=$ Shadow length of the wall
$\mathrm{H}=$ Height of Object casting shadow
D = Distance from the wall to the obstruction
$\theta$ = Sun Altitude Angle


Figure B: Procedure for Determining Height of shadow up a rooftop RSH $=(S H-W H) \times$ sine $A_{1}$ sine $A_{2}$

Where:
RSH = Roof shadow height
SH = Shadow height at Wall (determined in equation 1)
WH = Wall Height
$\mathrm{A}_{1}=90^{\circ}$ - Solar Altitude
$A_{2}=90^{\circ}-A_{1}+$ roof pitch


Figure C: Establishing Height Limits for Trees to Protect Rooftop Solar Access
$H=X+\left(D_{1}+D_{2}\right) \operatorname{Tan} \theta$

## Where

H = Height of Object Casting Shadow
X = Height from ground to bottom of collector
$\mathrm{D}_{1}=$ Distance from property line to the collector
$D_{2}=$ Distance from property line to Tree
$\Theta=$ Sun Altitude Angle


To determine the latitude and longitude for your municipality, see the attached summary table that identifies these parameters by zip code areas within the region. The attached table also provides the sun's altitude on December $21^{\text {st }}$ at 12 Noon solar time since this is the worst day of the year for access to solar energy. The shadow cast by a onemeter object is presented for each zip code area in the region and this information can be used to estimate the shadows cast by larger objects located to the south of a solar energy collector.

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## Latitude and Longitude of Municipalities in Western Connecticut by Zip Code

| ZIP Code | Municipality | State | Latitude <br> (North) | Longitude <br> (West) | Solar Altitude on December 21st (degree/ minutes) | Solar <br> Noon | Shadow Cast by 1 meter pole Solar Noon December 21st on flat land (Meters) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 06801 | Bethel | CT | 41.3759 | 73.3933 | 25.19 | 11:51:37 | 2.13 |
| 06752 | Bridgewater | CT | 41.5211 | 73.3597 | 23.03 | 11:51:29 | 2.14 |
| 06804 | Brookfield | CT | 41.4668 | 73.3928 | 25.09 | 11:51:40 | 2.14 |
| 06810 | Danbury | CT | 41.3768 | 73.4601 | 25.17 | 11:51:51 | 2.13 |
| 06811 | Danbury | CT | 41.4236 | 73.4845 | 25.14 | 11:52:00 | 2.13 |
| 06820 | Darien | CT | 41.0804 | 73.4823 | 25.49 | 11:51:53 | 2.1 |
| 06830 | Greenwich | CT | 41.0502 | 73.6235 | 25.55 | 11:52:31 | 2.09 |
| 06831 | Greenwich | CT | 41.0864 | 73.6612 | 25.55 | 11:52:41 | 2.09 |
| 06870 | Greenwich | CT | 41.2234 | 73.3353 | 25.55 | 11:52:17 | 2.09 |
| 06878 | Greenwich | CT | 41.5364 | 73.3517 | 25.54 | 11:52:23 | 2.09 |
| 06807 | Greenwich | CT | 41.4641 | 73.3644 | 25.54 | 11:52:28 | 2.09 |
| 06840 | New Canaan | CT | 41.1589 | 73.4989 | 25.42 | 11:52:00 | 2.1 |
| 06812 | New Fairfield | CT | 41.4862 | 73.4974 | 25.11 | 11:51:58 | 2.13 |
| 06776 | New Milford | CT | 41.6202 | 73.4053 | 24.99 | 11:51:40 | 2.15 |
| 06470 | Newtown | CT | 41.3932 | 73.3201 | 25.15 | 11:51:15 | 2.13 |
| 06850 | Norwalk | CT | 41.1272 | 73.4433 | 25.45 | 11:51:40 | 2.1 |
| 06851 | Norwalk | CT | 41.1388 | 73.4037 | 25.42 | 11:51:37 | 2.1 |
| 06853 | Norwalk | CT | 41.0695 | 73.4379 | 25.51 | 11:51:38 | 2.1 |
| 06854 | Norwalk | CT | 41.0941 | 73.4328 | 25.48 | 11:51:42 | 2.1 |
| 06855 | Norwalk | CT | 41.1001 | 73.3971 | 25.47 | 11:51:36 | 2.1 |
| 06856 | Norwalk | CT | 41.6145 | 73.2455 | 25.47 | 11:51:41 | 2.1 |
| 06877 | Ridgefield | CT | 41.3064 | 73.5024 | 25.29 | 11:52:01 | 2.12 |
| 06896 | Redding | CT | 41.3054 | 73.393 | 25.27 | 11:51:33 | 2.12 |
| 06784 | Sherman | CT | 41.5795 | 73.4985 | 24.99 | 11:52:00 | 2.15 |
| 06901 | Stamford | CT | 41.0531 | 73.5379 | 25.52 | 11:52:11 | 2.09 |
| 06902 | Stamford | CT | 41.061 | 73.5493 | 25.5 | 11:52:13 | 2.1 |
| 06903 | Stamford | CT | 41.1356 | 73.571 | 25.43 | 11:52:12 | 2.1 |
| 06904 | Stamford | CT | 41.0537 | 73.539 | 25.51 | 11:52:10 | 2.1 |
| 06905 | Stamford | CT | 41.0876 | 73.5444 | 25.51 | 11:52:09 | 2.1 |
| 06906 | Stamford | CT | 41.0697 | 73.522 | 25.5 | 11:52:07 | 2.1 |
| 06907 | Stamford | CT | 41.1005 | 73.521 | 25.47 | 11:52:06 | 2.1 |
| 06883 | Weston | CT | 41.2268 | 73.373 | 25.37 | 11:51:33 | 2.11 |
| 06880 | Westport | CT | 41.1454 | 73.3462 | 2.543 | 11:51:27 | 2.1 |
| 06897 | Wilton | CT | 41.207 | 73.4401 | 25.37 | 11:51:46 | 2.11 |

For precise latitude and longitude for your site go to the following:
https://www.esrl.noaa.gov/gmd/grad/solcalc/
https://keisan.casio.com/exec/system/1224682331

